**Mathematics**

**Problem 1: Topology of Moduli Spaces.**
How does the topology of moduli $M_k(X)$ of vector bundles changes under birational transformations of the base $X$?

**Theorem 1: Homology of Moduli Spaces.**
Let $Z$ be a compact surface containing a curve $C$ with $C^2 = -n$ and let $X$ be obtained from $Z$ by contracting the curve $C$. Then
$$H_2(M_k(Z), \Omega_k(X)) \neq 0.$$  
If $n$ is even, then also
$$H_1(M_k(Z), \Omega_k(X)) \neq 0.$$  

**Theorem 2: Atiyah Jones conjecture for rational surfaces.**
For any rational surface $X$, and $q < k/2$
$$H_q(M_k(X)) = H_q(M_{k+1}(X)).$$

**Problem 2: Classification of singularities.**
The classical invariants are not fine enough to distinguish inequivalent curve singularities.

**Theorem 3: Instanton numbers of singularities.**
Instanton numbers are finer than classical invariants.

<table>
<thead>
<tr>
<th>Singularities</th>
<th>Instantons $j = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>polynomial</td>
<td>$m$ $\delta_p$ $\mu$ $\tau$ $w$ $h$ charge</td>
</tr>
<tr>
<td>$x^3 - x^2 y + y^2$</td>
<td>3 3 4 4 4 3 7</td>
</tr>
<tr>
<td>$x^3 - x^2 y^2 + y^3$</td>
<td>3 3 4 4 5 3 8</td>
</tr>
</tbody>
</table>

**Translation**

**Mathematics $\rightarrow$ Physics**

Kobayashi – Hitchin correspondence: holomorphic bundles $\Leftrightarrow$ Instantons

Moduli space of bundles with first Chern number 0 and second Chern number $k$ $\Leftrightarrow$ Moduli space of instantons of charge $k$.

**Methods**

Surgery on holomorphic bundles $\Rightarrow$ Instanton “decay”

Construction of instantons $\Rightarrow$ Construction of sheaves

**Physics $\rightarrow$ Mathematics**

**Theorem 1': Instantons and topology of the base manifold.**
How are instantons affected by the contraction of a spherical body?

**Theorem 2': Stability of instanton moduli.**
Instanton moduli have topological stability under increase of charge (homology in low degrees is maintained).

**Theorem 3': Instantons detect singularities.**
Instanton numbers provide fine invariants for curve singularities.

**Open questions**

1. Stability of moduli in higher dimensions?
2. Relative homology of moduli of bundles on threefolds?
3. New invariants for hypersurface singularities? (the instanton methods used for curves readily generalizes to higher dimensions).
4. Physical interpretation of obstruction to instanton decay.

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